

# Supply Chain Coordination with Option Contract and Demand Information Asymmetry

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**Abstract:** This paper deals with the coordination of supply chain composed of a leading supplier and one retailer as a follower under asymmetric demand information. The demand information asymmetry is portrayed by the state of demand forecast information, accordingly the model, the state of demand information is the special discrete distribution, is established. The parameters menus, which are the optimal option price, the optimal option exercise price and transfer payment, are determined. The effect of demand information asymmetry on the efficiency of supply chain coordination is studied by the comparison with that of information symmetry. The results indicate that the option contract can coordinate the supply chain under demand information asymmetry; the supplier can nearly eliminate the influence of information asymmetric through the option menu parameters' design and extract all the channel profit but only leave the retailer the reservation profit.

**Keywords:** demand information asymmetry; option contract; supply chain coordination; Bayesian Nash Equilibrium

## I. Introduction

The supply chain is a distributional system which is composed by the independent decision-makers, the information asymmetrical situation also generally exists in operation process of supply chain, the decision-makers act according to the information situation and the individual benefit maximization as the goal to make the corresponding decision-making, the way of this disperse decision-making leads to the reduction of the supply chain coordination efficiency, which don't take the overall benefit maximization as a goal. Therefore, the key of raising the supply chain coordination efficiency is to sharing trustworthy information undoubtedly, the massive literature from this angle studied the contract design to motivate the upstream and downstream of supply chain sharing information, which can reduce the effect of information asymmetry to the supply chain coordination.

The contract design research under the information asymmetric condition of the supply chain may divide into two kinds:①information screening, this is called in the game theory that the information non-gaining side provides the contract to the information acquisition side who possessed the greater preponderance of information to gain the

information by the time;②signal gambling, the information acquisition side causes the information non-gain side through the contract to believe firmly the information is trustworthy [1]. This paper makes the conduct of the contract research from the information screening angle, the research results one hand are summarized as followed. The current research on the information asymmetrical situation are involving cost structure[2][3], demand information[4][5] and so on, the contract forms are used including quantity discount contract[6], wholesale price contract[4] and buy-back contract[7] etc. However, the majority of contracts are adopted in the enforced way and the option contract is merely employed when supply chain coordination is studied under the demand information asymmetry. Because the option contract as financial instruments has the different characters with the contract form above-mentioned, it is significant to study the supply chain coordination under the demand information asymmetry condition with voluntary enforced option contract.

## II. Problem Description and Symbol Hypothesis

Considering a two-echelon supply chain constituted by a supplier and a retailer, it produces and sales a product with a long delivery lead time, high product costs, and short selling season and price falling quickly and random demand  $D$ . In the production and sale process, it is assumed that the supplier as a leader and retailers as a follower are risk-neutral and fully rational, that is, both are expected to make decisions on the principle of profit maximization. As the retailer is closer to the market, so he has information superiority over the supplier, and the market demand information for retailers is considered to have two states, a low demand  $D_L$  and a high demand  $D_H$ . The supplier knows the probability of the market demand for  $D_L$  is  $p$ , and the probability of the market demand for  $D_H$  is  $(1-p)$ . Other symbols will be utilized by this paper are defined as follows:  $D_i$  ( $i=H, L$ ), the random demand, it possesses two states,  $D_H$  and  $D_L$ ;  $F_i(x)$  and  $f_i(x)$ , CDFs and PDFs of the two demand states;  $w_{ei}$  and  $w_{oi}$ , the option execution price and the option price when the demand state is  $D_i$ ;  $T_i$ , the transfer payment when the demand state is  $D_i$ ;  $Q_{ij}$ , the option order quantity under demand information state  $j$  while the real demand information is state  $i$ ;  $c$  and  $r$ , the production cost per unit and the sale price per unit;  $\Pi_i^r$  and  $\Pi_i^s$ , the profit function of

the retailer and supplier when the real demand information is state  $i$ .

To simplify without considering the shortage cost, inventory cost, and salvage value of unsold products. We assumed  $F_i(x)$  is increasing monotonically and the second-order is differentiable, and  $F_L(x) \geq F_H(x)$ ,  $x \geq 0$ , in order to reflect the high demand is higher random than low demand. Without loss of generality, we assume  $r > w_{ei} > w_{oi}$ .

### III. The Coordination Model with Option Contract and Demand Information Asymmetry

According to game theory, first the retailer establishes the optimal strategy, and then the supplier makes his own decisions in line with the retailer's strategy.

#### The retailer's decisions

Retailer first considers the two menus of option contract parameters,  $(w_{eH}, w_{oH}, T_H)$  and  $(w_{eL}, w_{oL}, T_L)$ , given by the supplier, and she has to make a choice and determines the option order quantity  $Q_{ij}$ . When the real demand information is state  $i$ , but the retailer chooses the contract type under demand information state  $j$ , her expected profit function is:

$$\Pi_i'(w_{ej}, w_{oj}, T_j, Q_{ij}) = (r - w_{ej}) \int_0^{Q_{ij}} \bar{F}_i(x) dx - w_{oj} Q_{ij} - T_j \quad (1)$$

The best option order quantity  $Q_{ij}^*$  can be attained by Eq. (1):

$$\bar{F}_i(Q_{ij}^*) = w_{oj} / (r - w_{ej}) \quad (2)$$

When the retailer's option order quantity satisfies Eq. (2), she can achieve the maximization of self-interest. The supplier develops this strategy of the retailer to design demand information sharing mechanism accordingly to achieve the supply chain coordination and minimize the impact of information asymmetry.

#### The supplier's decisions

When the real demand information is state  $i$ , but the retailer chooses the contract type under demand information state  $j$ , the supplier's expected profit function is:

$$\Pi_i^s(w_{ej}, w_{oj}, T_j, Q_{ij}) = w_{ej} \int_0^{Q_{ij}} \bar{F}_i(x) dx + (w_{oj} - c) Q_{ij} + T_j \quad (3)$$

The supplier's decision can be described as the following linear programming problem in line with the revelation principle and his expected profit maximization.

$$\max \Pi^s = p \Pi_L^s(w_{eL}, w_{oL}, T_L) + (1-p) \Pi_H^s(w_{eH}, w_{oH}, T_H) \quad (4)$$

$$\text{s.t. } \Pi_L^r(w_{eL}, w_{oL}, T_L, Q_{LL}) \geq 0 \quad (5)$$

$$\Pi_H^r(w_{eH}, w_{oH}, T_H, Q_{HH}) \geq 0 \quad (6)$$

$$\Pi_L^r(w_{eL}, w_{oL}, T_L, Q_{LL}) \geq \Pi_L^r(w_{eH}, w_{oH}, T_H, Q_{LH}) \quad (7)$$

$$\Pi_H^r(w_{eH}, w_{oH}, T_H, Q_{HH}) \geq \Pi_H^r(w_{eL}, w_{oL}, T_L, Q_{HL}) \quad (8)$$

Eq. (4) is the object function of the supplier; Eq. (5-6) are participation constraints to ensure that the retailer is retained the reservation profit to accept option contract; Eq. (7-8) are incentive compatibility constraints to ensure that the contract

type coincides with the real demand information state to achieve the requirements of the trusted information sharing. In order to facilitate the following reasoning, first Lemma 1 and its proof (see Appendix 1) are given.

**Lemma 1**  $F_L(\cdot)$  and  $f_L(\cdot)$ ,  $F_H(\cdot)$  and  $f_H(\cdot)$  are CDFs and PDFs under two states of demand information,  $x \geq 0$ ,  
 $F_L(x) \geq F_H(x)$ ,  $F_L(0) \geq F_H(0)$ , we assume  
 $n(x) = F_H^{-1}(F_L(x))$ ,

The function,

$$m(x) = \int_0^{n(x)} x f_H(x) dx - \int_0^x x f_L(x) dx, \text{ is nonnegative and}$$

monotone increasing when  $x \in [0, +\infty]$ .

**Theorem 1** The constraints Eq. (5-8) can be transformed into the two following Eq. (9-10). (Proof in Appendix 2)

$$T_L = (r - w_{eL}) \int_0^{Q_{LL}} x f_L(x) dx \quad (9)$$

$$T_H = (r - w_{eH}) \int_0^{Q_{HH}} x f_H(x) dx - (r - w_{eL}) \left[ \int_0^{Q_{HH}} x f_H(x) dx - \int_0^{Q_{LL}} x f_L(x) dx \right] \quad (10)$$

The supplier's expected profit function can be expressed by Eq. (1) and Eq. (9-10) as follow:

$$\begin{aligned} \Pi^s &= p \Pi_L^s(w_{eL}, w_{oL}, T_L) + (1-p) \Pi_H^s(w_{eH}, w_{oH}, T_H) \\ &= p \left[ r \int_0^{Q_{LL}} \bar{F}_L(x) dx - c Q_{LL} \right] + (1-p) \left\{ r \int_0^{Q_{HH}} \bar{F}_H(x) dx - c Q_{HH} \right. \\ &\quad \left. - (r - w_{eL}) \left[ \int_0^{Q_{HH}} x f_H(x) dx - \int_0^{Q_{LL}} x f_L(x) dx \right] \right\} \quad (11) \end{aligned}$$

In order to meet the requirements of the supplier's expected profit maximization, Eq. (11) is carried out first-order partial derivatives on  $w_{eL}$ ,  $w_{eH}$ ,  $Q_{HH}$ ,  $Q_{LL}$ .

$$\frac{\partial \Pi^s}{\partial w_{eL}} = (1-p) \left[ \int_0^{Q_{HH}} x f_H(x) dx - \int_0^{Q_{LL}} x f_L(x) dx \right] \geq 0 \quad (12)$$

$$\frac{\partial \Pi^s}{\partial w_{eH}} = 0 \quad (13)$$

$$\frac{\partial \Pi^s}{\partial Q_{LL}} = p [r \bar{F}_L(Q_{LL}) - c] - (1-p)(r - w_{eL})(Q_{HL} - Q_{LL}) f_L(Q_{LL}) \quad (14)$$

$$\frac{\partial \Pi^s}{\partial Q_{HH}} = (1-p) [r \bar{F}_H(Q_{HH}) - c] \quad (15)$$

We set arbitrary small positive number  $\varepsilon$ ,  $w_{eL}^* = r - \varepsilon$  is known from Eq. (12); from Eq. (13), we know the value of  $w_{eH}^*$  is not unique, its rang of values is  $0 < w_{eH}^* < r$ ; from Eq. (14), we can get  $p [r \bar{F}_L(Q_{LL}^*) - c] - (1-p) \varepsilon (Q_{HL}^* - Q_{LL}^*) f_L(Q_{LL}^*)$  and  $Q_{HL}^* = \bar{F}_H^{-1}(Q_{LL}^*)$ ,  $Q_{LL}^*$  satisfies these two equations above; from Eq. (14), we can get  $Q_{HH}^* = \bar{F}_H^{-1}(c/r)$ . Taking into account of Eq. (3) and the above analysis,  $w_{oL}^* = \varepsilon \bar{F}_L^{-1}(Q_{LL}^*)$  and  $w_{oH}^* = (r - w_{eH}^*) \frac{c}{r}$  are yielded. It is easy to work out the best transfer fee  $T_H^*$  and  $T_L^*$ . Finally,

we summarized the parameters design the supplier's option contract as Theorem 2.

**Theorem 2** As the demand information asymmetry, the supplier by the design of option contract can encourage the retailer to share real demand information and the option contract can (almost) completely coordinate the supply chain.

i) On the condition of the demand information is the low state, the option contract parameters are:

$$w_{eL}^* = r - \varepsilon; \quad w_{oL}^* = (r - w_{eH}^*) \bar{F}_L^{-1}(Q_{LL}^*);$$

$$T_L^* = (r - w_{eL}^*) \int_0^{Q_{LL}^*} x f_L(x) dx$$

ii) On the condition of the demand information is the high state, the option contract parameters are:

$$0 < w_{eH}^* \leq w_{eL}^*; \quad w_{oH}^* = (r - w_{eH}^*) \frac{c}{r};$$

$$T_H^* = (r - w_{eH}^*) \int_0^{Q_{HH}^*} x f_H(x) dx - (r - w_{eL}^*) \left[ \int_0^{Q_{HL}^*} x f_H(x) dx - \int_0^{Q_{LL}^*} x f_L(x) dx \right]$$

We can conclude that  $w_{eH}^* \leq w_{eL}^*$  and  $T_H^* \geq T_L^*$  from Theorem 2, that is, the option contract has higher transfer fees and lower option execution price, and vice versa. It is easily known that optimal order quantity of the supply chain of centralized decision-making is  $Q_c^* = \bar{F}_i^{-1}(c/r)$ . We can get  $Q_{HH}^* = \bar{F}_H^{-1}(c/r) = Q_c^*$  and  $Q_{LL}^* \rightarrow \bar{F}_L^{-1}(c/r) = Q_c^*$  when  $\varepsilon \rightarrow 0$  from Eq. (14-15). So the option contract can completely coordinate the supply chain when the demand information state is high and the option contract can almost coordinate the supply chain.

The presence of asymmetric information is beneficial to the information superiority side; the retailer can obtain the opportunity profit by sharing false demand information, while the supplier bears the information rent. We form the following theorem 3 and show that the supplier can almost eliminate this impact of information asymmetry.

**Theorem 3** when  $\varepsilon \rightarrow 0$ , opportunity profit the retailer obtains or information rent the supplier bears are:

$$p \Pi_L'(w_{eL}^*, w_{oL}^*, T_L^*) + (1-p) \Pi_H'(w_{eH}^*, w_{oH}^*, T_H^*) \\ = \varepsilon(1-p) \left[ \int_0^{Q_{HL}^*} x f_H(x) dx - \int_0^{Q_{LL}^*} x f_L(x) dx \right] \rightarrow 0$$

(Proof in Appendix 3)

## IV. The Conclusion

In the environment of asymmetric demand information exists between the supplier and the retailer, the states of demand information are employed to portray the asymmetry, the coordination model is built with option contract and demand information asymmetry, we prove that the model can (almost) completely coordinate the supply chain and eliminate this impact of information asymmetry. Although information sharing can usually improve the channel profits of supply chain, but our result indicates that the retailer possessed of demand information superiority can make use of opportunistic behavior and seeks to maximize their own

profit, the supplier can design the option contract to achieve trusted information sharing and the all channel profit is nearly obtained by the supplier, the retailer does not benefit from the information sharing.

In the current uncertain market environment, the information asymmetry has a vital influence on decisions of supply chain coordination. This paper analyzes a simple structure of supply chain with demand information asymmetry, in which how the inferior information side as the leader incentive the follower to achieve a genuine information sharing and can obtain all channel profit, so this study has some significance in practice. This paper can further consider the supplier has a certain capacity constraint, and can also be extended to the existence of competition in supply chain.

## Appendices

### Appendix 1

From the hypothesis, we have  $F_H[n(x)] = F_L(x) \geq F_H(x)$ .  $F_H(x)$  is increasing in its domain of definition, so  $n(x) \geq x$ .

Because  $F_H'[n(x)]n'(x) = f_L(x)$  and  $m(0) = 0$ , therefore

$$m'(x) = n'(x)$$

$f_H[n(x)]n(x) - x f_L(x) \geq 0$ , thus  $m(x)$  is nonnegative and increasing monotonically in  $x \in [0, +\infty]$ .

### Appendix 2

Firstly, from Eq. (5) and Eq. (8) we can prove Eq. (6)

$$\begin{aligned} & \Pi_H'(w_{eH}, w_{oH}, T_H, Q_{HH}) \\ &= (r - w_{eH}) \int_0^{Q_{HH}} x f_H(x) dx - T_H \\ &\geq (r - w_{eL}) \int_0^{Q_{HL}} x f_H(x) dx - T_L \quad (\text{From Eq. (8)}) \\ &\geq (r - w_{eL}) \left[ \int_0^{Q_{HL}} x f_H(x) dx - \int_0^{Q_{LL}} x f_L(x) dx \right] \quad (\text{From Eq. (5)}) \\ &\geq 0 \quad (\text{From Lemma 1}) \end{aligned}$$

Secondly, we can get Eq. (16-17) from Eq. (5), Eq. (8) and the analysis above,

$$\begin{aligned} T_L &\leq (r - w_{eL}) \int_0^{Q_{LL}} x f_L(x) dx \quad (16) \\ T_H &\leq (r - w_{eH}) \int_0^{Q_{HH}} x f_H(x) dx - (r - w_{eL}) \left[ \int_0^{Q_{HL}} x f_H(x) dx - \int_0^{Q_{LL}} x f_L(x) dx \right] \quad (17) \end{aligned}$$

From Theorem 3, we know that the supplier is urgent to reduce the information rent arising from the information asymmetry as much as possible, besides improper setting of  $T_H$  and  $T_L$  may cause the opportunism behavior of the retailer. Considering two aspects above, we obtain Theorem 1.

Finally, it must be mentioned that Eq. (7) can be proved when the best option contract parameters are obtained.

$$\begin{aligned} & \Pi_L'(w_{eL}, w_{oL}, T_L, Q_{LL}) - \Pi_H'(w_{eH}, w_{oH}, T_H, Q_{HH}) \\ &= (r - w_{eL}) \int_0^{Q_{LL}} x f_L(x) dx - T_L - (r - w_{eH}) \int_0^{Q_{HL}} x f_H(x) dx + T_H \end{aligned}$$

$$\begin{aligned}
&= (r - w_{eH}) \int_0^{Q_{HH}} x f_H(x) dx - (r - w_{eL}) \left[ \int_0^{Q_{HL}} x f_H(x) dx - \int_0^{Q_{LL}} x f_L(x) dx \right] \\
&\quad - (r - w_{eH}) \int_0^{Q_{LH}} x f_L(x) dx \quad (\text{From Eq.(9-10)}) \\
&= (r - w_{eH}) \left[ \int_0^{Q_{HH}} x f_H(x) dx - \int_0^{Q_{HL}} x f_L(x) dx \right] \\
&\quad - (r - w_{eL}) \left[ \int_0^{Q_{HH}} x f_H(x) dx - \int_0^{Q_{LL}} x f_L(x) dx \right] \\
&\geq (r - w_{eL}) \left[ \left( \int_0^{Q_{HH}} x f_H(x) dx - \int_0^{Q_{LL}} x f_L(x) dx \right) \right. \\
&\quad \left. + \left( \int_0^{Q_{HL}} x f_H(x) dx - \int_0^{Q_{LH}} x f_L(x) dx \right) \right]
\end{aligned}$$

From Lemma 1, we know that

$$\begin{aligned}
&\int_0^{Q_{HH}} x f_H(x) dx - \int_0^{Q_{LL}} x f_L(x) dx \geq 0 \\
&\int_0^{Q_{HL}} x f_H(x) dx - \int_0^{Q_{LH}} x f_L(x) dx \geq 0
\end{aligned}$$

thus

$$\Pi_L^r(w_{eL}, w_{oL}, T_L, Q_{LL}) - \Pi_L^r(w_{eH}, w_{oH}, T_H, Q_{LH}) \geq 0$$

that is,

$$\Pi_L^r(w_{eL}, w_{oL}, T_L, Q_{LL}) \geq \Pi_L^r(w_{eH}, w_{oH}, T_H, Q_{LH}).$$

### Appendix 3

$$\begin{aligned}
&p \Pi_L^r(w_{eL}^*, w_{oL}^*, T_L^*) + (1-p) \Pi_H^r(w_{eH}^*, w_{oH}^*, T_H^*) \\
&= p \left[ (r - w_{eL}^*) \int_0^{Q_{LL}^*} x f_L(x) dx - T_L^* \right] \\
&\quad + (1-p) \left[ (r - w_{eH}^*) \int_0^{Q_{HH}^*} x f_H(x) dx - T_H^* \right] \quad (\text{From Eq.(3)}) \\
&= p \left[ (r - w_{eL}^*) \int_0^{Q_{LL}^*} x f_L(x) dx - (r - w_{eL}^*) \int_0^{Q_{LL}^*} x f_L(x) dx \right] \\
&\quad + (1-p) \left\{ (r - w_{eH}^*) \int_0^{Q_{HH}^*} x f_H(x) dx - (r - w_{eH}^*) \int_0^{Q_{HH}^*} x f_H(x) dx \right. \\
&\quad \left. + (r - w_{eL}^*) \left[ \int_0^{Q_{HL}^*} x f_H(x) dx - \int_0^{Q_{LL}^*} x f_L(x) dx \right] \right\} \quad (\text{From Eq.(9-10)}) \\
&= \varepsilon (1-p) \left[ \int_0^{Q_{HL}^*} x f_H(x) dx - \int_0^{Q_{LL}^*} x f_L(x) dx \right] \rightarrow 0
\end{aligned}$$

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